

CERN-PH-TH/2010-118

CPHT-RR037.0510

LPT-ORSAY 10-32

NSF-KITP-10-057

Non-linear MSSM

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Abstract

Using the formalism of constrained superfields, we derive the most general effective action of a light goldstino coupled to the minimal supersymmetric standard model (MSSM) and study its phenomenological consequences. The goldstino-induced couplings become important when the (hidden sector) scale of spontaneous supersymmetry breaking, \sqrt{f} , is relatively low, of the order of few TeV. In particular, we compute the Higgs potential and show that the (tree level) mass of the lightest Higgs scalar can be increased to the LEP bound for $\sqrt{f} \sim 2$ TeV to 7 TeV. Moreover, the effective quartic Higgs coupling is increased due to additional tree-level contributions proportional to the ratio of visible to hidden sector supersymmetry breaking scales. This increase can alleviate the amount of fine tuning of the electroweak scale that exists in the MSSM. Among the new goldstino couplings, beyond those in MSSM, the most important ones generate an invisible decay of the Higgs boson into a goldstino and neutralino (if $m_h > m_{\chi_1^0}$), with a partial decay rate that can be comparable to the SM channel $h^0 \rightarrow \gamma\gamma$. A similar decay of Z boson is possible if $m_Z > m_{\chi_1^0}$ and brings a lower bound on \sqrt{f} that must be of about 700 GeV. Additional decay modes of the Higgs or Z bosons into a pair of light goldstinos, while possible, are suppressed by an extra $1/f$ factor and have no significant impact on the model.

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1 Introduction

Spontaneous supersymmetry breaking at low energies predicts a nearly massless goldstino. More precisely, it plays the role of the longitudinal component of the gravitino, which acquires a Planck suppressed mass f/M_{Planck} , in the milli-eV range if the supersymmetry breaking scale \sqrt{f} is in the multi-TeV region. By the equivalence theorem [1], it interacts with a strength $1/\sqrt{f}$ which is much stronger than the Planck suppressed couplings of the transverse gravitino, and is therefore very well described by the gravity-decoupled limit of a massless Goldstone fermion. An example of such a situation is provided by gauge mediation where, however, the typical scale of supersymmetry breaking is expected to be a few orders of magnitude higher than the soft breaking terms of the Standard Model (SM) superparticles, due to their double suppression by the loop factor and by the messengers mass.

In this work, we perform a model independent analysis of the low energy consequences of a light goldstino by treating \sqrt{f} as a free parameter, that can be as low as a few times the scale of soft breaking terms which we denote generically m_{soft} . Furthermore, we will assume that all extra states (that may exist beyond those of the MSSM) are heavier than \sqrt{f} . In such a framework, there are two generic energy regimes that can be studied: **(i)**: at TeV energies, comparable (or higher) than m_{soft} , one has the usual MSSM together with a goldstino; **(ii)**: at low energies, lower than all sparticle masses, one is left just with a goldstino coupled to the SM fields. In both cases, the goldstino effective interactions can be determined by non-linear supersymmetry. In the first case, it couples to ordinary supermultiplets of linear supersymmetry, while in the second case the superparticles have been integrated out.

The self-interactions of the goldstino are given by the famous Volkov-Akulov action [2]. Their geometric method gives also a universal coupling to matter through its energy momentum tensor, of the form $(1/f^2)T_{\mu\nu}t^{\mu\nu}$, where $T_{\mu\nu}$, $t_{\mu\nu}$ are the stress tensors of matter and of (free) goldstino, respectively [3, 4]. It was realized however that this coupling is not the most general invariant under non-linear supersymmetry [4, 5, 6]. General invariant couplings can be derived using two different superfield formulations. One of them promotes any ordinary field to a superfield by introducing a modified superspace that takes into account the non-linear supersymmetry transformations of the goldstino [7, 8, 9]. The other uses the formalism of constrained superfields: these are usual superfields, but are subject to constraints that eliminate the superpartners in terms of the light degrees of freedom and the goldstino [10, 11, 12, 13].

In this work, we use the method of constrained superfields in order to determine the general couplings of the goldstino to MSSM superfields, focusing in the first energy region mentioned above $E \sim m_{soft} < \sqrt{f}$. The only constrained superfield is then that of the goldstino X_{nl} , satisfying the constraint $X_{nl}^2 = 0$, which couples to MSSM superfields via the corresponding

soft terms: the recipe is to replace the spurion $S \equiv m_{soft}\theta^2$ by $(m_{soft}/f)X_{nl}$ and solve for its F-auxiliary component, as usual, in order to determine all effective interactions that can be expanded in inverse power series of f . In the second energy region, lower than m_{soft} , the superpartners can be integrated out (this can be done by additional constraints on the MSSM superfields [13]), and one has the goldstino coupled to SM fields only. In this case, it was found that the dominant effective operators are of dimension-six [9] and can induce an important invisible decay width of the Higgs boson, if the goldstino carries lepton number [14]. For a related effective approach to these problems, goldstino couplings and applications see [15, 16].

Obviously, the goldstino couplings to MSSM become important if the supersymmetry (SUSY) breaking scale is low. On the other hand, validity of the effective Lagrangian requires that f be higher than the soft breaking terms, so that m_{soft}^2/f is a good expansion parameter. It turns out that the most important effects of these couplings are in the Higgs sector. In particular, the quartic Higgs coupling is increased by a term proportional to the ratio of visible to hidden sector SUSY breaking, with two important consequences: (i) it can increase the tree-level value of the lightest Higgs mass that can then reach and cross the LEP bound² of 114.4 GeV [22]; (ii) it can alleviate the fine tuning of the electroweak scale in MSSM due to the relatively high experimental bounds on m_{soft} and large quantum corrections usually required in MSSM to satisfy the LEP bound. Additional effects that we investigate relate to the goldstino-induced couplings in the MSSM Lagrangian, upon integration of the sgoldstino. All couplings goldstino - MSSM fields are computed and these can be used for phenomenological studies. As an example we show that for a light neutralino, the SM-like Higgs can decay into a goldstino (which is the lightest supersymmetric particle (LSP)) and the lightest neutralino (next to LSP (NLSP) in this case), with a decay rate that can be comparable to the SM partial decay $h^0 \rightarrow \gamma\gamma$. A similar decay of Z is possible, which provides a lower bound on $\sqrt{f} \approx 700$ GeV. Other decays of the Higgs and Z bosons into pairs of goldstinos are possible, but they have additional $(1/f)$ suppression, with little impact on the allowed parameter space.

The paper is organized as follows. Section 2 discusses general issues in non-linear realizations of supersymmetry and goldstino couplings. Section 3 presents the “non-linear” MSSM model obtained by the general coupling of the MSSM to goldstino, using the method described above. Section 4 presents the new couplings of the model, not present in MSSM, some of which are dimension-four in fields, suppressed by up to the second power of $1/f$. Section 5 analyzes the implications for the Higgs masses. Section 6 presents other phenomenological consequences, such as the implications for the fine-tuning of the electroweak scale, some interesting limits, and the invisible decays of the Higgs and Z bosons together with their constraints. Finally, Section 7 contains our concluding remarks.

²For other possibilities to increase the Higgs mass in effective models see [15, 17, 18, 19, 21].

2 Non-linear realizations and constrained goldstino superfield

An important role in constructing a non-linear supersymmetric version of the MSSM is played by the goldstino chiral superfield (SM gauge singlet) X_{nl} . One can use the component fields formalism to describe the corresponding Volkov-Akulov action. However, one can use the more convenient superfield formalism, endowed with constraints; for the goldstino superfield this constraint is $X_{nl}^2 = 0$ [10, 11, 12, 13]. One can start with the Lagrangian

$$\mathcal{L}_X = \int d^4\theta X^\dagger X + \left\{ \int d^2\theta f X + h.c. \right\} = |\partial_\mu \phi_X|^2 + F_X^\dagger F_X + \left[\frac{i}{2} \bar{\psi}_X \bar{\sigma}^\mu \partial_\mu \psi_X + f F_X + h.c. \right] \quad (1)$$

with the aforementioned constraint. This constraint is solved by

$$X_{nl} = \phi_X + \sqrt{2} \theta \psi_X + \theta \theta F_X, \quad \text{with} \quad \phi_X = \frac{\psi_X \psi_X}{2 F_X} \quad (2)$$

which, when used in eq.(1) recovers [13] the Volkov-Akulov Lagrangian. After using the equations of motion $F_X = -f + \dots$ where f (that can be chosen real) is the hidden sector SUSY breaking scale. Therefore, in the infrared description of the SUSY breaking (which is model independent), the scalar component (sgoldstino) becomes a function of the goldstino.

To find the goldstino couplings to matter fields (discussed for the MSSM in the next section), consider first a supersymmetric theory with chiral multiplets $\Phi_i \equiv (\phi_i, \psi_i, F_i)$ and vector multiplets $V \equiv (A_\mu, \lambda, D)$ coupled in a general way to X_{nl} :

$$\begin{aligned} \mathcal{L} = & \int d^4\theta \left[X_{nl}^\dagger X_{nl} + \Phi_i^\dagger (e^V \Phi)_i - (m_i^2/f^2) X_{nl}^\dagger X_{nl} \Phi_i^\dagger (e^V \Phi)_i \right] + \left\{ \int d^2\theta \left[f X_{nl} + W(\Phi_i) \right. \right. \\ & \left. \left. + \frac{B_{ij}}{2f} X_{nl} \Phi_i \Phi_j + \frac{A_{ijk}}{6f} X_{nl} \Phi_i \Phi_j \Phi_k + \frac{1}{4} \left(1 + \frac{2m_\lambda}{f} X_{nl} \right) \text{Tr} W^\alpha W_\alpha \right] + h.c. \right\}, \end{aligned} \quad (3)$$

where m_i^2, B_{ij}, A_{ijk} are soft terms for the scalars and m_λ is the gaugino mass. From this, one can find the Goldstino (ψ_X) couplings to ordinary matter/gauge superfields. These couplings can be checked to be equivalent to those obtained by the equivalence theorem [1], from a theory with the corresponding explicit soft breaking, in which the goldstino couples as:

$$(1/f) \partial^\mu \psi_X J_\mu = -(1/f) \psi_X \partial^\mu J_\mu + (\text{total space-time derivative}), \quad (4)$$

Here J_μ is the supercurrent of the theory corresponding to that in (3) in which the goldstino is essentially replaced by the spurion, with the corresponding explicit soft breaking terms:

$$\begin{aligned} \mathcal{L}' = & \int d^4\theta \left[1 - m_i^2 \theta^2 \bar{\theta}^2 \right] \Phi_i^\dagger (e^V \Phi)_i + \int d^2\theta \left[W(\Phi_i) - (1/2) B_{ij} \theta^2 \Phi_i \Phi_j - (1/6) A_{ijk} \theta^2 \Phi_i \Phi_j \Phi_k \right. \\ & \left. + \frac{1}{4} (1 - 2m_\lambda \theta^2) \text{Tr} W^\alpha W_\alpha \right] + h.c. , \end{aligned} \quad (5)$$

With this, eq.(4) shows that, on-shell, all goldstino couplings are proportional to soft terms. Indeed, the supercurrent of (5) is given by (with $\mathcal{D}_{\mu,ij} = \delta_{ij} \partial_\mu + i g A_\mu^a T_{ij}^a$)

$$J_\alpha^\mu = -[\sigma^\nu \bar{\sigma}^\mu \psi_i]_\alpha [\mathcal{D}_{\nu,ij} \phi_j]^\dagger + i [\sigma^\mu \bar{\psi}_i]_\alpha F_i - \frac{1}{2\sqrt{2}} [\sigma^\nu \bar{\sigma}^\rho \sigma^\mu \bar{\lambda}^a]_\alpha F_{\nu\rho}^a + \frac{i}{\sqrt{2}} D^a [\sigma^\mu \bar{\lambda}^a]_\alpha \quad (6)$$

so

$$\partial_\mu J_\alpha^\mu = \psi_{i,\alpha} (m_i^2 \phi_j^\dagger + B_{ij} \phi_j + (1/2) A_{ijk} \phi_j \phi_k) + \frac{m_\lambda}{\sqrt{2}} [(\sigma^{\mu\nu})_\alpha^\beta \lambda_\beta^a F_{\mu\nu}^a + D^a \lambda_\alpha^a] . \quad (7)$$

From (4), (7) one then recovers the couplings with one goldstino. However, the superfield formalism in (3) has the advantage that is easier to use when evaluating couplings with more than one goldstino, by simply writing all effective operators (involving X_{nl}) to a fixed order in $1/f$ [13]. It is more difficult to find these from (5) and in Section 3 we use the former method.

Finally, in addition to usual SUSY and goldstino couplings eq.(3) also brings new goldstino-independent couplings induced by eliminating F_X . Indeed, from (3)

$$\left(1 - \frac{m_i^2}{f^2} |\phi_i|^2\right) F_X^\dagger = -\left(f + \frac{B_{ij}}{2f} \phi_i \phi_j + \frac{A_{ijk}}{6f} \phi_i \phi_j \phi_k + \frac{m_\lambda}{2f} \lambda \lambda + \dots\right), \quad (8)$$

So $|F_X|^2$ generates new couplings in onshell \mathcal{L} , such as quartic scalar terms. When applied to MSSM, this brings in particular new corrections to the Higgs scalar potential (see later).

3 The “non-linear” MSSM.

We apply the above method to couple the constrained superfield X_{nl} to the SUSY part of the MSSM, to find the “non-linear” supersymmetry version of MSSM [13]. We stress that at energy scales below m_{soft} , similar constraints can be applied to the MSSM superfields themselves, corresponding to integrating out the corresponding superpartners. Here, the only difference from the ordinary MSSM is in the supersymmetry breaking sector. Supersymmetry is broken spontaneously via a vacuum expectation value (VEV) of F_X , fixed by its equation of motion (see later). The Lagrangian of the “non-linear MSSM” model is [13],

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_X + \mathcal{L}_H + \mathcal{L}_m + \mathcal{L}_{AB} + \mathcal{L}_g \quad (9)$$

Let us detail these terms. \mathcal{L}_0 is the usual MSSM SUSY Lagrangian, in standard notation:

$$\begin{aligned} \mathcal{L}_0 = & \sum_{\Phi, H_{1,2}} \int d^4\theta \Phi^\dagger e^{V_i} \Phi + \left\{ \int d^2\theta \left[\mu H_1 H_2 + H_2 Q U^c + Q D^c H_1 + L E^c H_1 \right] + h.c. \right\} \\ & + \sum_{\text{SM groups}} \frac{1}{16 g^2 \kappa} \int d^2\theta \text{Tr} [W^\alpha W_\alpha] + h.c., \quad \Phi : Q, D^c, U^c, E^c, L, \end{aligned} \quad (10)$$

κ is a constant canceling the trace factor and the gauge coupling g is shown explicitly.

The SUSY breaking couplings originate from the MSSM fields couplings to the goldstino superfield; this is done by the replacement $S \rightarrow (1/f) m_{soft} X_{nl}$ [13], where S is the spurion, with $S = \theta\theta m_{soft}$ and m_{soft} is a generic notation for the soft terms (denoted below $m_{1,2}, B, m_0$). One has for the Higgs sector

$$\begin{aligned}
\mathcal{L}_H &= \sum_{i=1,2} c_i \int d^4\theta X_{nl}^\dagger X_{nl} H_i^\dagger e^{V_i} H_i \\
&= \sum_{i=1,2} c_i \left\{ |\phi_X|^2 \left[|\mathcal{D}_\mu h_i|^2 + F_{h_i}^\dagger F_{h_i} + h_i^\dagger \frac{D_i}{2} h_i + \left(\frac{i}{2} \bar{\psi}_{h_i} \bar{\sigma}^\mu \mathcal{D}_\mu \psi_{h_i} - \frac{1}{\sqrt{2}} h_i^\dagger \lambda_i \psi_{h_i} + h.c. \right) \right] \right. \\
&\quad + \frac{1}{2} h_i^\dagger (\mathcal{D}_\mu + \overleftarrow{\mathcal{D}}_\mu) h_i \partial^\mu |\phi_X|^2 + \bar{\psi}_X \bar{\psi}_{h_i} \psi_X \psi_{h_i} - \frac{1}{2} [\phi_X^\dagger (\partial^\mu - \overleftarrow{\partial}^\mu) \phi_X] [h_i^\dagger (\mathcal{D}_\mu - \overleftarrow{\mathcal{D}}_\mu) h_i] \\
&\quad + \left[-\frac{i}{2} \phi_X^\dagger \psi_X \sigma^\mu \bar{\psi}_{h_i} (\mathcal{D}_\mu - \overleftarrow{\mathcal{D}}_\mu) h_i - \frac{1}{\sqrt{2}} \phi_X^\dagger \psi_X h_i^\dagger \lambda_i h_i - \phi_X^\dagger \psi_X F_{h_i}^\dagger \psi_{h_i} + \phi_X^\dagger F_X F_{h_i}^\dagger h_i \right. \\
&\quad + \frac{i}{2} (\bar{\psi}_X \bar{\sigma}^\mu \psi_X) (h_i^\dagger \mathcal{D}_\mu h_i) + \frac{i}{2} (\phi_X^\dagger \partial_\mu \phi_X) (\bar{\psi}_{h_i} \bar{\sigma}^\mu \psi_{h_i}) + \frac{i}{2} \bar{\psi}_X \bar{\sigma}^\mu (\partial_\mu - \overleftarrow{\partial}_\mu) \phi_X (h_i^\dagger \psi_{h_i}) \\
&\quad \left. - \bar{\psi}_X F_X \bar{\psi}_{h_i} h_i + h.c. \right] + \left[\partial_\mu \phi_X^\dagger \partial^\mu \phi_X + F_X^\dagger F_X + \left(\frac{i}{2} \bar{\psi}_X \bar{\sigma}^\mu \partial_\mu \psi_X + h.c. \right) \right] |h_i|^2 \Big\}, \quad (11)
\end{aligned}$$

Here $\mathcal{D}, \partial, (\overleftarrow{\mathcal{D}}, \overleftarrow{\partial})$ act only on the first field to their right (left) respectively and h_i, ψ_{h_i}, F_{h_i} denote SU(2) doublets. Also

$$c_1 = -m_1^2/f^2, \quad c_2 = -m_2^2/f^2. \quad (12)$$

Similar terms exist for all matter fields Q, U^c, D^c, L, E^c :

$$\mathcal{L}_m = \sum_\Phi c_\Phi \int d^4\theta X_{nl}^\dagger X_{nl} \Phi^\dagger e^V \Phi, \quad c_\Phi = -\frac{m_\Phi^2}{f^2}, \quad \Phi : Q, U^c, D^c, L, E^c, \quad (13)$$

One can eventually set $m_\Phi = m_0$ (all Φ). The bi- and trilinear SUSY breaking couplings are

$$\begin{aligned}
\mathcal{L}_{AB} &= \frac{B}{f} \int d^2\theta X_{nl} H_1 H_2 \\
&\quad + \frac{A_u}{f} \int d^2\theta X_{nl} H_2 Q U^c + \frac{A_d}{f} \int d^2\theta X_{nl} Q D^c H_1 + \frac{A_e}{f} \int d^2\theta X_{nl} L E^c H_1 + h.c. \\
&= \frac{B}{f} \left\{ \phi_X \left[h_1 \cdot F_{h_2} + F_{h_1} \cdot h_2 - \psi_{h_1} \cdot \psi_{h_2} \right] - h_1 \cdot (\psi_X \psi_{h_2}) - (\psi_X \psi_{h_1}) \cdot h_2 + F_X h_1 \cdot h_2 \right\} \\
&\quad + \left\{ \frac{A_u}{f} \left[\phi_X h_2 \cdot (\phi_Q F_U - \psi_Q \psi_U + F_Q \phi_U) - \phi_X (\psi_{h_2} \cdot \phi_Q \psi_U + \psi_{h_2} \cdot \psi_Q \phi_U - F_{h_2} \cdot \phi_Q \phi_U) \right. \right. \\
&\quad - \psi_X (h_2 \cdot \phi_Q \psi_U + h_2 \cdot \psi_Q \phi_U + \psi_{h_2} \cdot \phi_Q \phi_U) + F_X h_2 \cdot \phi_Q \phi_U \Big] - \left[U \rightarrow D, H_2 \rightarrow H_1 \right] \\
&\quad \left. - \left[U \rightarrow E, H_2 \rightarrow H_1, Q \rightarrow L \right] \right\} + h.c. \quad (14)
\end{aligned}$$

where $B \equiv B_0 m_0 \mu$. For simplicity, Yukawa matrices are not displayed; to recover them just replace above and in formulae below any pair of fields $\phi_Q \phi_U \rightarrow \phi_Q \gamma_u \phi_U$, $\phi_Q \phi_D \rightarrow \phi_Q \gamma_d \phi_D$, $\phi_L \phi_E \rightarrow \phi_L \gamma_e \phi_E$; similar for the fermions and auxiliary fields, with $\gamma_{u,d,e}$ 3×3 matrices.

Finally, the supersymmetry breaking couplings in the gauge sector are

$$\begin{aligned}
\mathcal{L}_g &= \sum_{i=1}^3 \frac{1}{16 g_i^2 \kappa} \frac{2 m_{\lambda_i}}{f} \int d^2 \theta X_{nl} \text{Tr} [W^\alpha W_\alpha]_i + h.c. \\
&= \sum_{i=1}^3 \frac{m_{\lambda_i}}{2 f} \left\{ \phi_X \left[2 i \lambda^a \sigma^\mu \Delta_\mu \bar{\lambda}^a - \frac{1}{2} F^{a \mu \nu} F_{\mu \nu}^a + D^a D^a - \frac{i}{4} \epsilon^{\mu \nu \rho \sigma} F_{\mu \nu}^a F_{\rho \sigma}^a \right] \right. \\
&\quad \left. - \sqrt{2} \psi_X \sigma^{\mu \nu} \lambda^a F_{\mu \nu}^a - \sqrt{2} \psi_X \lambda^a D^a + F_X \lambda^a \lambda^a \right\}_i + h.c. \quad (15)
\end{aligned}$$

with $m_{\lambda_{1,2,3}}$ the masses of the three gauginos and gauge group index i for $U(1)$, $SU(2)$, $SU(3)$ respectively. Above we introduced the notation $\Delta_\mu \bar{\lambda}^a = \partial_\mu \bar{\lambda}^a - g t^{abc} V_\mu^b \bar{\lambda}^c$. Equations (1) to (15) define the model, with spontaneous supersymmetry breaking ensured by non-zero $\langle F_X \rangle$.

Since $\phi_X \sim 1/f$, the Lagrangian contains terms of order higher than $1/f^2$. In the calculation of the onshell Lagrangian we shall restrict the calculations to up to and including $1/f^2$ terms. This requires solving for F_ϕ of matter fields up to and including $1/f^2$ terms and for F_X up to and including $1/f^3$ terms (due to its leading contribution which is $-f$). Doing so, in the final Lagrangian no kinetic mixing is present at this order. Using the expressions of the auxiliary fields, one then computes the F -part of the scalar potential of the Higgs sector, to find:

$$V_F = |\mu|^2 \left[|h_1|^2 + |h_2|^2 \right] + \frac{|f + (B/f) h_1 \cdot h_2|^2}{1 + c_1 |h_1|^2 + c_2 |h_2|^2} + \mathcal{O}(1/f^3) \quad (16)$$

with $h_1 \cdot h_2 \equiv h_1^0 h_2^0 - h_1^- h_2^+$ and $|h_1|^2 \equiv h_1^\dagger h_1 = h_1^{0*} h_1^0 + h_1^{-*} h_1^-$, etc. One can work with this potential, however, for convenience, if $|c_{1,2}| |h_{1,2}|^2 \ll 1$, we can approximate V_F by expanding the denominator in a series of powers of these coefficients. Our analysis below is then valid for $|c_{1,2}| |h_{1,2}|^2 \ll 1$. After adding the gauge contribution, we find the following result for the scalar potential of the Higgs sector:

$$\begin{aligned}
V &= f^2 + (|\mu|^2 + m_1^2) |h_1|^2 + (|\mu|^2 + m_2^2) |h_2|^2 + (B h_1 \cdot h_2 + h.c.) \\
&\quad + \frac{1}{f^2} \left[m_1^2 |h_1|^2 + m_2^2 |h_2|^2 + B h_1 \cdot h_2 \right]^2 + \frac{g_1^2 + g_2^2}{8} \left[|h_1|^2 - |h_2|^2 \right]^2 + \frac{g_2^2}{2} |h_1^\dagger h_2|^2 + \mathcal{O}(1/f^3)
\end{aligned} \quad (17)$$

This is the full Higgs potential. The first term in the last line is a new term, absent in MSSM (generated by eliminating F_X of X_{nl}). Its effects for phenomenology will be analyzed later. The ignored higher order terms in $1/f$ involve non-renormalizable $h_{1,2}^6$ interactions in V .

4 New couplings in the Lagrangian.

In this section we compute the new interactions induced by Lagrangian (9), which are not present in the MSSM. Many of the new couplings are actually dimension-four in fields, with a (dimensionless) f -dependent coupling. The couplings are important in the case of a low SUSY breaking scale in the hidden-sector and a light gravitino scenario. Some of the new couplings also involve the goldstino field and are relevant for phenomenology.

As mentioned earlier in Section 3, from the SUSY breaking part of the Lagrangian only terms up to and including $1/f^2$ were kept in the total Lagrangian given by equations (1) to (15). After eliminating all terms proportional to F -auxiliary fields of $X, H_i, Q, D^c, U^c, E^c, L$, one obtains new couplings \mathcal{L}^{new} beyond those of the usual *onshell*, *supersymmetric* part of MSSM, which are unchanged and not shown. One finds the onshell Lagrangian

$$\mathcal{L}^{new} \equiv \mathcal{L}_F^{aux} + \mathcal{L}_D^{aux} + \mathcal{L}_m^{extra} + \mathcal{L}_g^{extra} \quad (18)$$

Let us detail these terms. Firstly,

$$\mathcal{L}_F^{aux} = \mathcal{L}_{F(1)}^{aux} + \mathcal{L}_{F(2)}^{aux} \quad (19)$$

with

$$\begin{aligned} \mathcal{L}_{F(1)}^{aux} = & - \left[f^2 + (m_1^2 |h_1|^2 + m_2^2 |h_2|^2 + m_\Phi^2 |\phi_\Phi|^2) \right] \\ & - \left[B h_1 \cdot h_2 + A_u h_2 \cdot \phi_Q \phi_U + A_d \phi_Q \phi_D \cdot h_1 + A_e \phi_L \phi_E \cdot h_1 + \frac{1}{2} m_{\lambda_i} \lambda_i \lambda_i + h.c. \right] \end{aligned} \quad (20)$$

recovering the usual MSSM soft terms and the additional contributions:

$$\begin{aligned} \mathcal{L}_{F(2)}^{aux} = & \left\{ \frac{\bar{\psi}_X \bar{\psi}_X}{2 f^2} \left[\mu (m_1^2 + m_2^2) h_1 \cdot h_2 - (m_1^2 + m_Q^2 + m_D^2) h_1 \cdot \phi_Q \phi_D - (m_1^2 + m_L^2 + m_E^2) h_1 \cdot \phi_L \phi_E \right. \right. \\ & - (m_2^2 + m_Q^2 + m_U^2) \phi_Q \phi_U \cdot h_2 + (B h_2 - A_d \phi_Q \phi_D - A_e \phi_L \phi_E)^\dagger (\mu h_2 - \phi_Q \phi_D - \phi_L \phi_E) \\ & + (B h_1 - A_u \phi_Q \phi_U)^\dagger (\mu h_1 - \phi_Q \phi_U) + (A_d \phi_D h_1 - A_u h_2 \phi_U)^\dagger (\phi_D h_1 - h_2 \phi_U) \\ & + A_d (|\phi_Q \cdot h_1|^2 + |\phi_E h_1|^2) + A_u |h_2 \cdot \phi_Q|^2 + A_e |\phi_L \cdot h_1|^2 \left. \right] + h.c. \left. \right\} - \frac{1}{f^2} \left| B h_1 \cdot h_2 \right. \\ & + A_u h_2 \cdot \phi_Q \phi_U + A_d \phi_Q \phi_D \cdot h_1 + A_e \phi_L \phi_E \cdot h_1 + \frac{m_{\lambda_i}}{2} \lambda_i \lambda_i + (m_1^2 |h_1|^2 + m_2^2 |h_2|^2 + m_\Phi^2 |\phi_\Phi|^2) \left. \right|^2 \\ & - \frac{1}{f} \left[m_1^2 \bar{\psi}_X \bar{\psi}_{h_1} h_1 + m_2^2 \bar{\psi}_X \bar{\psi}_{h_2} h_2 + m_\Phi^2 \bar{\psi}_X \bar{\psi}_\Phi \phi_\Phi + h.c. \right] + \mathcal{O}(1/f^3) \end{aligned} \quad (21)$$

A summation is understood over the SM group indices: $i = 1, 2, 3$ in the gaugino term and over $\Phi = Q, U^c, D^c, L, E^c$ in the mass terms; appropriate contractions among $SU(2)_L$ doublets

are understood for holomorphic products, when the order displayed is relevant. The leading interactions $\mathcal{O}(1/f)$ are those in the last line and are dimension-four in fields. Similar couplings exist at $\mathcal{O}(1/f^2)$ and involve scalar and gaugino fields. Yukawa matrices are restored in (21) by replacing $\phi_Q \phi_D \rightarrow \phi_Q \gamma_d \phi_D$, $\phi_Q \phi_U \rightarrow \phi_Q \gamma_u \phi_U$, $\phi_L \phi_E \rightarrow \phi_L \gamma_e \phi_E$, as already explained.

There are also new couplings from terms involving the auxiliary components of the vector superfields of the SM. Integrating them out one finds:

$$\begin{aligned} \mathcal{L}_D^{aux} = & \frac{-1}{2} \left[\tilde{D}_1 + \frac{1}{4f^2} (m_{\lambda_1} \psi_X \psi_X + h.c.) \tilde{D}_1 + \frac{1}{\sqrt{2}f} (m_{\lambda_1} \psi_X \lambda_1 + h.c.) \right]^2 \\ & + \frac{-1}{2} \left[\tilde{D}_2^a + \frac{1}{4f^2} (m_{\lambda_2} \psi_X \psi_X + h.c.) \tilde{D}_2^a + \frac{1}{\sqrt{2}f} (m_{\lambda_2} \psi_X \lambda_2^a + h.c.) \right]^2 \\ & + \frac{-1}{2} \left[\tilde{D}_3^a + \frac{1}{4f^2} (m_{\lambda_3} \psi_X \psi_X + h.c.) \tilde{D}_3^a + \frac{1}{\sqrt{2}f} (m_{\lambda_3} \psi_X \lambda_3^a + h.c.) \right]^2 + \mathcal{O}(1/f^3) \end{aligned} \quad (22)$$

with the notation:

$$\begin{aligned} \tilde{D}_1 &= -\frac{1}{2} g_1 (-h_1^\dagger h_1 + h_2^\dagger h_2 + 1/3 \phi_Q^\dagger \phi_Q - 4/3 \phi_U^\dagger \phi_U + 2/3 \phi_D^\dagger \phi_D - \phi_L^\dagger \phi_L + 2 \phi_E^\dagger \phi_E) \\ \tilde{D}_2^a &= -\frac{1}{2} g_2 (h_1^\dagger \sigma^a h_1 + h_2^\dagger \sigma^a h_2 + \phi_Q^\dagger \sigma^a \phi_Q + \phi_L^\dagger \sigma^a \phi_L) \\ \tilde{D}_3^a &= -\frac{1}{2} g_3 (\phi_Q^\dagger t^a \phi_Q - \phi_U^\dagger t^a \phi_U - \phi_D^\dagger t^a \phi_D) \end{aligned} \quad (23)$$

for the MSSM corresponding expressions; here $(t^a/2)$ are the $SU(3)$ generators. From (22) one can easily read the new, f -dependent couplings in the gauge sector, absent in the MSSM.

The total Lagrangian also contains extra terms, not proportional to the auxiliary fields, and *not* present in the MSSM. In the matter sector these are:

$$\begin{aligned} \mathcal{L}_m^{extra} = & \frac{1}{4f^2} |\partial_\mu (\psi_X \psi_X)|^2 + \left(\frac{i}{2} \bar{\psi}_X \bar{\sigma}^\mu \partial_\mu \psi_X + h.c. \right) \\ & - \sum_{i=1}^2 \frac{m_i^2}{f^2} \left\{ \bar{\psi}_X \bar{\psi}_{h_i} \psi_X \psi_{h_i} + \left[\frac{i}{2} (\bar{\psi}_X \bar{\sigma}^\mu \psi_X) (h_i^\dagger \mathcal{D}_\mu h_i) + \frac{i}{2} |h_i|^2 \bar{\psi}_X \bar{\sigma}^\mu \partial_\mu \psi_X + h.c. \right] \right\} \\ & - \left[m_i^2 \rightarrow m_\Phi^2, H_i \rightarrow \Phi \right] + \left\{ \frac{B}{f} \left[\frac{1}{2f} \psi_X \psi_X \psi_{h_1} \cdot \psi_{h_2} - h_1 \cdot (\psi_X \psi_{h_2}) - (\psi_X \psi_{h_1}) \cdot h_2 \right] \right. \\ & + \frac{A_u}{f} \left[\frac{1}{2f} \psi_X \psi_X (h_2 \cdot \psi_Q \psi_U + \psi_{h_2} \cdot \phi_Q \psi_U + \psi_{h_2} \cdot \psi_Q \phi_U) - \psi_X (h_2 \cdot \phi_Q \psi_U + h_2 \cdot \psi_Q \phi_U \right. \\ & + \left. \left. \psi_{h_2} \cdot \phi_Q \phi_U) \right] + \left[\frac{A_d}{f} \left(\frac{1}{2f} \psi_X \psi_X (\psi_Q \psi_D \cdot h_1 + \phi_Q \psi_D \cdot \psi_{h_1} + \psi_Q \phi_D \cdot \psi_{h_1}) \right. \right. \\ & - \left. \left. \psi_X (\phi_Q \psi_D \cdot h_1 + \psi_Q \phi_D \cdot h_1 + \phi_Q \phi_D \cdot \psi_{h_1}) \right) + (D \rightarrow E, L \rightarrow Q) \right] + h.c. \left. \right\} + \mathcal{O}(1/f^3). \end{aligned} \quad (24)$$

Note the presence of interactions that are dimension-four in fields ($B/f h_1 \psi_X \psi_{h_2}$, etc) that can be relevant for phenomenology at low f . There are also new couplings in the gauge sector

$$\begin{aligned}
\mathcal{L}_g^{extra} &= \sum_{i=1}^3 \frac{m_{\lambda_i}}{2f} \left[\frac{\psi_X \psi_X}{-2f} \left(2i \lambda^a \sigma^\mu \Delta_\mu \bar{\lambda}^a - \frac{1}{2} F_{\mu\nu}^a F^{a\mu\nu} - \frac{i}{4} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a \right) \right. \\
&\quad \left. - \sqrt{2} \psi_X \sigma^{\mu\nu} \lambda^a F_{\mu\nu}^a \right]_i + h.c. + \mathcal{O}(1/f^3),
\end{aligned} \tag{25}$$

with $i = 1, 2, 3$ the gauge group index and $\sigma^{\mu\nu} = i/4 (\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu)$. The new couplings of \mathcal{L}^{new} together with the *onshell* part of the purely *supersymmetric* part of the MSSM Lagrangian (onshell \mathcal{L}_0 of (10)) gives the final onshell effective Lagrangian of the model. From this, the full scalar potential is identified.

5 Implications for the Higgs masses.

Let us consider the Higgs scalar potential found in (17) and analyze the implications for the Higgs masses. From the neutral Higgs part of potential one finds the masses of the CP even and CP odd Higgs fields. Exact values (in $1/f$) can be found (see the Appendix), but since eq. (17) is valid up to $1/f^4$ terms, it is sufficient to present the expressions of the Higgs masses that are valid up to this order. Firstly, at the minimum of the scalar potential one has:

$$\begin{aligned}
m_1^2 - m_2^2 &= \cot 2\beta \left[B + \frac{f^2}{v^2} \frac{(-1 + \sqrt{w_0})(-B + m_Z^2 \sin 2\beta)}{2\mu^2 + m_Z^2 \cos^2 2\beta + B \sin 2\beta} \right] \\
m_1^2 + m_2^2 &= \frac{1}{\sin 2\beta} \left[-B + \frac{f^2}{v^2} \frac{(-1 + \sqrt{w_0})(B + 2\mu^2 \sin 2\beta)}{2\mu^2 + m_Z^2 \cos^2 2\beta + B \sin 2\beta} \right]
\end{aligned} \tag{26}$$

where

$$w_0 \equiv 1 - \frac{v^2}{f^2} (4\mu^2 + 2m_Z^2 \cos^2 2\beta + 2B \sin 2\beta) \tag{27}$$

There is a second solution for $m_{1,2}^2$ at the minimum (with minus in front of $\sqrt{w_0}$) which however is not a perturbation of the MSSM one and not considered below (since it brings a shift proportional to f of the soft masses, which invalidates the expansion in $m_{1,2}^2/f$). One finds the following results (upper sign for m_h^2 and lower sign for m_H^2):

$$\begin{aligned}
m_{h,H}^2 &= \frac{1}{2} \left[m_Z^2 + \frac{-2B}{\sin 2\beta} \mp \sqrt{w_1} \right] + \frac{v^2}{32f^2} \left\{ 4B \left[2B + (4\mu^2 + 2m_Z^2 \cos^2 2\beta)/\sin 2\beta \right] \right. \\
&\quad + 4 \left[2B^2 + 8\mu^4 + 2m_Z^2(4\mu^2 + m_Z^2) \cos^2 2\beta + 8B\mu^2 \sin 2\beta \right] \\
&\quad \mp \frac{\csc^2 2\beta}{\sqrt{w_1}} \left[-2(B^2 + 4\mu^4)m_Z^2 + 4\mu^2 m_Z^4 + m_Z^6 + 8(2\mu^4 m_Z^2 - B^2(4\mu^2 + m_Z^2)) \cos 4\beta \right. \\
&\quad - m_Z^2(6B^2 + 8\mu^4 + 4\mu^2 m_Z^2 + m_Z^4) \cos 8\beta - 8B(B^2 - 8\mu^4) \sin 2\beta \\
&\quad \left. \left. + B(-8B^2 + 16\mu^2 m_Z^2 + m_Z^4) \sin 6\beta + Bm_Z^4 \sin 10\beta \right] \right\} + \mathcal{O}(1/f^3)
\end{aligned} \tag{28}$$

with

$$w_1 = \left(m_Z^2 + \frac{-2B}{\sin 2\beta}\right)^2 - 4m_Z^2 \left(\frac{-2B}{\sin 2\beta}\right) \cos^2 2\beta \quad (29)$$

Further, the mass m_A of the pseudoscalar Higgs has a simple form (no expansion):

$$m_A^2 = \frac{-2B}{\sin 2\beta} \left\{ \frac{3}{4} + \frac{1}{4} \sqrt{w_0} - \frac{v^2}{4f^2} B \sin 2\beta \right\} \quad (30)$$

and, as usual, the Goldstone mode has mass $m_G = 0$.

It is instructive to consider the limit of large $u \equiv \tan \beta$, with $B < 0$ fixed, when

$$m_h^2 = \left[m_Z^2 + \mathcal{O}(1/u)\right] + \frac{v^2}{2f^2} \left[(2\mu^2 + m_Z^2)^2 + \frac{4}{u} B (2\mu^2 + m_Z^2) + \mathcal{O}(1/u^2)\right] + \mathcal{O}(1/f^3) \quad (31)$$

$$m_H^2 = \left[\frac{-2B}{\sin 2\beta} + \mathcal{O}(1/u)\right] + \frac{v^2 B}{4f^2} \left[(2\mu^2 + m_Z^2)u + 4B + \frac{1}{u}(2\mu^2 - 11m_Z^2) + \mathcal{O}(1/u^2)\right] + \mathcal{O}(1/f^3)$$

which shows that a large μ can increase m_h (decrease m_H). However, for phenomenology it is customary to use m_A as an input instead of B , in which case the masses $m_{h,H}$ take the form

$$\begin{aligned} m_{h,H}^2 = & \frac{1}{2} \left[m_A^2 + m_Z^2 \mp \sqrt{w} \right] \pm \frac{v^2}{16f^2} \frac{1}{\sqrt{w}} \left[16m_A^2 \mu^4 + 4m_A^2 \mu^2 m_Z^2 + (m_A^2 - 8\mu^2) m_Z^4 \right. \\ & - 2m_Z^6 \pm 2(-2m_A^2 \mu^2 + 8\mu^4 + 4\mu^2 m_Z^2 + m_Z^4) \sqrt{w} + m_A^2 m_Z^4 \cos 8\beta \\ & + m_A^4 (m_A^2 - 8\mu^2 - 3m_Z^2) \sin^2 2\beta + \cos 4\beta \left[-2m_Z^2 (8\mu^4 + 4\mu^2 m_Z^2 + m_Z^4 - m_A^2 (6\mu^2 + m_Z^2)) \right. \\ & \left. \left. \pm 2(2m_A^2 \mu^2 + 4\mu^2 m_Z^2 + m_Z^4) \sqrt{w} - m_A^2 (m_A^2 + 5m_Z^2) \sin^2 2\beta \right] \right] + \mathcal{O}(1/f^3) \end{aligned} \quad (32)$$

where the first term (bracket) is just the MSSM contribution. The upper (lower) signs correspond to m_h (m_H) and $w = (m_A^2 + m_Z^2)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta$. At large $\tan \beta$ with m_A fixed one finds³ (with $u \equiv \tan \beta$)

$$\begin{aligned} m_h^2 &= \left[m_Z^2 + \mathcal{O}(1/u^2) \right] + \frac{v^2}{2f^2} \left[(2\mu^2 + m_Z^2)^2 + \mathcal{O}(1/u^2) \right] + \mathcal{O}(1/f^3) \\ m_H^2 &= \left[m_A^2 + \mathcal{O}(1/u^2) \right] + \frac{1}{f^2} \mathcal{O}(1/u^2) + \mathcal{O}(1/f^3) \end{aligned} \quad (33)$$

In this limit the increase of m_h is driven by a large μ and apparently is of SUSY origin, but the quartic Higgs couplings giving this effect involved combinations of soft masses (see (17)). These soft masses combined to give, at the EW minimum, the μ -dependent increase in (33)⁴.

³ In (33) $m_A > m_Z$ is assumed, otherwise just exchange m_h^2 with m_H^2 .

⁴ See also λ of (34) evaluated at EW minimum, $\delta = 0$, $\tan \beta \rightarrow \infty$: $\lambda \rightarrow (1/2v^2)[m_Z^2 + v^2(m_Z^2 + 2\mu^2)/(2f^2)]$.

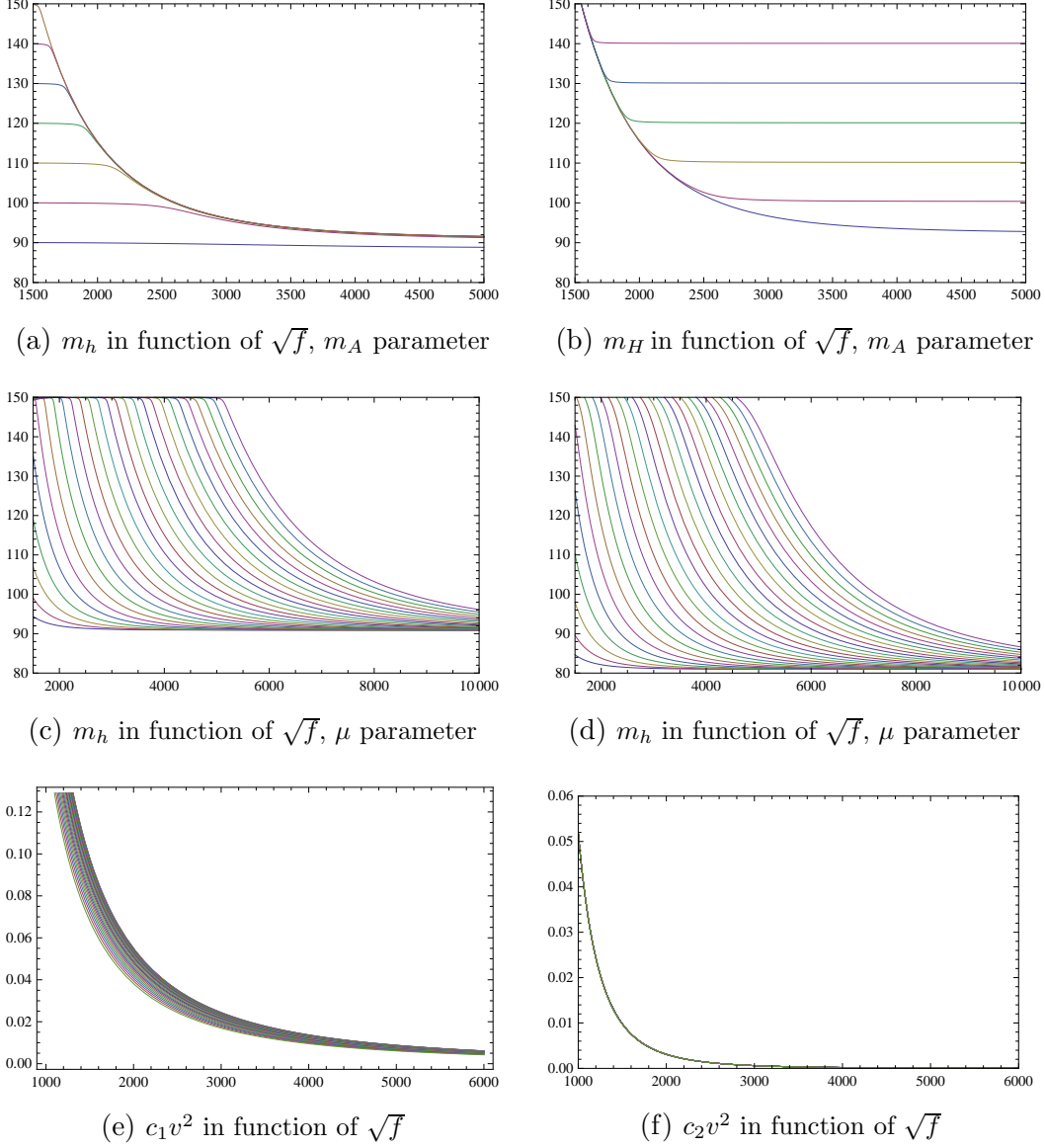


Figure 1: The tree-level Higgs masses (in GeV) and expansion coefficients as functions of \sqrt{f} (in GeV). In (a), (b) $\mu = 900$ GeV, $\tan \beta = 50$, m_A increases upwards from 90 to 150 GeV in steps of 10 GeV. The increase of m_h is significant even at larger \sqrt{f} , if one increases μ , as seen in (c), (d). In figs. (c), (d), $m_A = 150$ GeV and m_h increases as μ varies from 400 to 3000 GeV in steps of 100 GeV. In (c) $\tan \beta = 50$ while in (d) $\tan \beta = 5$, showing a milder dependence on $\tan \beta$ than in MSSM. For $\tan \beta \geq 10$ there is little difference from (c). In (e), (f) the expansion coefficients are shown, for $m_A = [90, 650]$ GeV with steps of 10 GeV, $\mu = 900$ GeV, $\tan \beta = 50$; they are always less than unity, even at larger values of \sqrt{f} or μ shown in (c), (d), as required for a convergent expansion.

Some simple numerical examples are relevant for the size of the corrections to the Higgs masses, relative to their MSSM values. The largest correction to m_h for large $\tan\beta$ is dominated by μ and f (eq. (33)). For example, if $(\mu/\sqrt{f})^2 = (1/2.25)^2 \approx 1/5$, $v = 246$ GeV, with $\mu = 900$ GeV then $\sqrt{f} = 2$ TeV, giving $m_h = 114.4$ GeV. Other examples are: ($\mu = 1.2$ TeV, $\sqrt{f} = 2.7$ TeV) and ($\mu = 2.6$ TeV, $\sqrt{f} = 6$ TeV), leading to $m_h = 114.4$ GeV (with $(\mu/\sqrt{f})^2 \approx 1/5$). Smaller $\mu \approx 600$ GeV can still allow m_h just above the LEP bound if $\sqrt{f} = 1.35$ TeV, for similar value for $(\mu/\sqrt{f})^2 = 1/5$ and for the rest of the parameters. This shows that one can have a *classical* value of m_h near or marginally above the LEP bound and larger than the classical MSSM value ($= m_Z$). The plots in Figure 1 illustrate better the value of m_h and m_H for various values of \sqrt{f} . For \sqrt{f} in the region of 1.5 TeV to 7 TeV the LEP bound is satisfied for m_h , while at larger \sqrt{f} the MSSM case is recovered. By varying \sqrt{f} our results can interpolate between low and high scale (in the hidden sector) SUSY breaking. Quantum corrections increase m_h further, just as in the MSSM.

Regarding the usual MSSM tree-level flat direction $|h_1^0| = |h_2^0|$ one can show that the potential in this direction can have a minimum for the case (not considered in MSSM) of $m_1^2 + m_2^2 + 2|\mu|^2 < 2|B|$, equal to $V_m = f^2 - (1/4)f^2(m_1^2 + m_2^2 + 2|\mu|^2 + 2B)^2/(m_1^2 + m_2^2 + B)^2$. Compared to the usual MSSM minimum, the former can be situated above it only for values of f which do not comply with the original assumptions of $m_{1,2}^2, |B| < f$. On the other hand, the case with V_m situated below the MSSM minimum does not allow one to recover the MSSM ground state in the decoupling limit of large f , and in conclusion the “flat” direction is not of physical interest here.

6 Other phenomenological implications.

6.1 Fine-tuning of the electroweak scale

The increase of m_h , at the classical level, beyond the MSSM tree-level bound (m_Z) and the presence of the new quartic couplings of the Higgs fields also have implications for the fine tuning. In the MSSM the smallness of the effective quartic coupling λ (fixed by the gauge sector) is at the origin of an increased amount of fine tuning of the electroweak scale for large soft masses. For soft masses significantly larger than the electroweak (EW) scale, (also needed to increase the MSSM value for m_h above LEP bound via quantum corrections), fine tuning increases rapidly⁵ and may become a potential problem (sometimes referred to as the “little hierarchy” problem). Let us see why in the present model this problem is alleviated. One can

⁵Two-loop MSSM fine tuning [23] is minimized at $m_h \sim 115$ GeV (consistent with EW and dark matter constraints); however, beyond this value, fine tuning increases *exponentially* with m_h .

write $v^2 = -m^2/\lambda$ where

$$\begin{aligned}\lambda &\equiv \frac{g_1^2 + g_2^2}{8} \left[\cos^2 2\beta + \delta \sin^4 \beta \right] + \frac{1}{f^2} \left| m_1^2 \cos^2 \beta + m_2^2 \sin^2 \beta + (1/2) B \sin 2\beta \right|^2 \\ m^2 &\equiv (|\mu|^2 + m_1^2) \cos^2 \beta + (|\mu|^2 + m_2^2) \sin^2 \beta + B \sin 2\beta\end{aligned}\quad (34)$$

The first term in λ is due to MSSM only, while the second one, which is positive, is due to the new quartic Higgs terms in (17). Here δ accounts for the top/stop quantum effects to $|h_2|^4$ term in the potential, which becomes $(1+\delta)(g_1^2 + g_2^2)/8 |h_2|^4$; usually $\delta \sim \mathcal{O}(1)$ (ignoring couplings other than top Yukawa). This quantum effect is only included for a comparison to the new quartic Higgs term. The important point to note is that a larger λ gives a suppression in the fine tuning measure Δ :

$$\Delta = \frac{\partial \ln v^2}{\partial \ln p} = \frac{\partial \ln(-m^2/\lambda)}{\partial \ln p}, \quad p = A, B, m_0^2, \mu^2, m_{\lambda_i}^2. \quad (35)$$

Here p is an MSSM parameter with respect to which fine tuning is evaluated. In the large $\tan \beta$ limit, the fine tuning of the electroweak scale becomes (see the Appendix in [19]):

$$\Delta = -\frac{(|\mu|^2 + m_2^2)'}{v^2 m_2^4/f^2 + (1+\delta) m_Z^2/2} + \mathcal{O}(1/\tan \beta), \quad (|\mu|^2 + m_2^2)' \equiv \frac{\partial(|\mu|^2 + m_2^2)}{\partial \ln p} \quad (36)$$

For small $\tan \beta$ a similar result is obtained in which one replaces m_2 by m_1 . The first term in denominator comes from the new correction to the effective quartic coupling λ . Larger soft masses $m_{1,2}$ increase λ and this can actually reduce fine tuning, see the denominator in Δ . Therefore, in this case heavier superpartners do not necessarily bring an increased fine tuning amount (as it usually happens in the MSSM). The only limitation here is the size of the ratio $m_{1,2}^2/f \leq 1$ for convergence of the nonlinear formalism. In the limit this coefficient approaches its upper limit (say $\sim 1/3$), the two contributions in the denominator have comparable size (for $\delta \sim 1$ and $v = 246$ GeV) and fine tuning is reduced by a factor ≈ 2 from that in the absence of the new term in the denominator (*i.e.* the MSSM case).

6.2 Limiting cases and loop corrections.

Some interesting limits of our “non-linear” MSSM model are worth considering. Firstly, in the limit of large f (*i.e.* large SUSY breaking scale in the hidden sector) and with $m_{1,2}, B$ *fixed*, the new quartic term in (17) vanishes, while the usual explicit soft SUSY breaking terms specific to the Higgs sector remain. This is just the MSSM case. All other couplings suppressed by inverse powers of f are negligible in this limit. Another limiting case is that of very small f . For our analysis to be valid, one needs to satisfy the condition $B, m_{1,2}^2 \leq f$. When f reaches this minimal bound, the new quartic couplings in (17), not present in the

MSSM, increase and eventually become closer to unity. The analysis is then less reliable and additional effective contributions in the Lagrangian, suppressed by higher powers like $1/f^4$ and beyond, may become relevant for SUSY breaking effects.

Finally, one remark regarding the calculation of radiative corrections using (17) and the electroweak symmetry breaking (EWSB). In our case EWSB was assumed to take place by appropriate values of $m_{1,2}^2, B$. However, the same EWSB mechanism as in the MSSM is at work here, via quantum corrections to these masses, which near the EW scale turn $m_2^2 + \mu^2$ negative and trigger radiative EWSB. Indeed, if the loops of the MSSM states are cut off as usual at the high GUT scale (well above \sqrt{f}) and with the new Higgs quartic couplings regarded as an *effective*, classical operator, radiative EWSB can take place as in the MSSM. A similar example is the case of a MSSM Higgs sector extended with additional effective operators of dimension $d = 5$ such as $(1/M) \int d^2\theta (H_1 H_2)^2$ giving a dimension-four (in fields) contribution to the scalar potential $V \supset (\mu/M) h_1 h_2 (|h_1|^2 + |h_2|^2)$; this is regarded as an effective operator and radiative EWSB is implemented as in the MSSM, see for example [18, 19]. The advantage in our case is that no “new physics” (scale M) is introduced in the visible sector. In both cases, the new scale M and our scale \sqrt{f} have comparable values, because in both cases the increase of m_h above the LEP bound is done via couplings depending on the ratio (μ/M) [17, 18, 19, 20, 21] or (μ/\sqrt{f}) , respectively.

It is interesting to remark that the loop corrections induced by the (effective) quartic couplings proportional to $1/f^2$ in eq.(17), can be under control at large f . Indeed, the loop integrals this coupling induces can be quadratically divergent and are then cut-off at momentum $p^2 \leq f$; but the loop effects come with a coupling factor that behaves like $1/f^2$, so overall they will be suppressed like $1/f$ and can then be under control even at large f (for a discussion of loop corrections involving the goldstino, see [24]).

6.3 Invisible decays of Higgs and Z bosons.

Let us analyze some implications of the interactions involving the goldstino field, described by the Lagrangian found above. An interesting possibility, for a light enough neutralino, is the decay of the neutral higgses into a goldstino and the lightest neutralino χ_1^0 (this is the NLSP, while goldstino is the LSP). The coupling Higgs-goldstino-neutralino is only suppressed by $1/f$. It arises from the following terms in \mathcal{L}^{new} and from the terms in the *onshell*, *SUSY* part of usual MSSM Lagrangian (10), hereafter denoted $\mathcal{L}_0^{onshell}$:

$$\begin{aligned} \mathcal{L}^{new} + \mathcal{L}_0^{onshell} \supset & -\frac{1}{f} \left[m_1^2 \psi_X \psi_{h_1^0} h_1^{0*} + m_2^2 \psi_X \psi_{h_2^0} h_2^{0*} \right] - \frac{B}{f} \left[\psi_X \psi_{h_2^0} h_1^0 + \psi_X \psi_{h_1^0} h_2^0 \right] \\ & - \frac{1}{f} \sum_{i=1,2} \frac{m_{\lambda_i}}{\sqrt{2}} \tilde{D}_i^a \psi_X \lambda_i^a - \frac{1}{\sqrt{2}} \left[g_2 \lambda_2^3 - g_1 \lambda_1 \right] \left[h_1^{0*} \psi_{h_1^0} - h_2^{0*} \psi_{h_2^0} \right] + h.c. \quad (37) \end{aligned}$$

The last term (present in the MSSM) also brings a goldstino interaction. This is possible through the goldstino components of the higgsinos $\psi_{h_{1,2}^0}$ and EW gauginos $\lambda_{1,2}$. The goldstino components are found via the equations of motion, after EWSB, to give (see also [13]):

$$\begin{aligned}\mu \psi_{h_1^0} &= \frac{1}{f\sqrt{2}} \left(-m_2^2 v_2 - B v_1 - \frac{1}{2} v_2 \langle g_2 D_2^3 - g_1 D_1 \rangle \right) \psi_X + \dots \\ \mu \psi_{h_2^0} &= \frac{1}{f\sqrt{2}} \left(-m_1^2 v_1 - B v_2 + \frac{1}{2} v_1 \langle g_2 D_2^3 - g_1 D_1 \rangle \right) \psi_X + \dots \\ \lambda_1 &= \frac{-1}{f\sqrt{2}} \langle D_1 \rangle \psi_X + \dots, \quad \lambda_2^3 = \frac{-1}{f\sqrt{2}} \langle D_2^3 \rangle \psi_X + \dots\end{aligned}\tag{38}$$

which can be further simplified by using the MSSM minimum conditions in the terms multiplied by $1/f$ (allowed in this approximation). As a consistency check we also showed that the determinant of the neutralino mass matrix (now a 5×5 matrix, to include the Goldstino) vanishes up to corrections of order $\mathcal{O}(f^{-4})$. This is consistent with our approximation for the Lagrangian, and verifies the existence of a massless Goldstino (ultimately “eaten” by the gravitino). Using (37) and (38), one finds after some calculations (for previous calculations of this decay see [25, 28, 29]):

$$\mathcal{L}^{new} + \mathcal{L}_0^{onshell} \supset -\frac{1}{f\sqrt{2}} \sum_{j,k=1}^4 \left[\psi_X \chi_j^0 H^0 \delta_k \mathcal{Z}_{jk}^* + \psi_X \chi_j^0 h^0 \delta'_k \mathcal{Z}_{jk}^* \right] + h.c.\tag{39}$$

where

$$\begin{aligned}\delta_1 &= m_Z \sin \theta_w [m_{\lambda_1} \cos(\alpha + \beta) + \mu \sin(\alpha - \beta)], \\ \delta_2 &= -m_Z \cos \theta_w [m_{\lambda_2} \cos(\alpha + \beta) + \mu \sin(\alpha - \beta)], \\ \delta_3 &= -m_A^2 \sin \beta \sin(\alpha - \beta) - \mu^2 \cos \alpha \\ \delta_4 &= m_A^2 \cos \beta \sin(\alpha - \beta) - \mu^2 \sin \alpha, \quad \delta'_i = \delta_i \Big|_{\alpha \rightarrow \alpha + \pi/2}\end{aligned}\tag{40}$$

\mathcal{Z} is the matrix that diagonalizes the MSSM neutralino mass matrix⁶: $M_d^2 = \mathcal{Z} M M^\dagger \mathcal{Z}^\dagger$, and can be easily evaluated numerically (see [27] for its analytical expression). Further H^0, h^0 are Higgs mass eigenstates (of mass $m_{h,H}$ computed earlier) and $h_i^0 = 1/\sqrt{2} (v_i + h_i^{0'} + i\sigma_i)$ with $\langle h_i^{0'} \rangle = 0$, $\langle \sigma_i \rangle = 0$; the relation of H^0, h^0 to $h_{1,2}^{0'}$ is a rotation, which in this case can be

⁶The exact form of M is: $M_{11} = m_{\lambda_1}$, $M_{12} = 0$, $M_{13} = -m_Z \cos \beta \sin \theta_w$, $M_{14} = m_Z \sin \beta \sin \theta_w$, $M_{21} = 0$, $M_{22} = m_{\lambda_2}$, $M_{23} = m_Z \cos \beta \cos \theta_w$, $M_{24} = -m_Z \sin \beta \cos \theta_w$, $M_{33} = 0$, $M_{34} = \mu$, $M_{44} = 0$, also $M_{ij} = M_{ji}$. Note the sign of μ related to our definition of the holomorphic product of $SU(2)$ doublets. With this notation, in the text $\chi_j^0 = \mathcal{Z}_{jk} \xi_k$, with $\xi_k^T \equiv (\lambda_1, \lambda_2^3, \psi_{h_1^0}, \psi_{h_2^0})$.

just that of the MSSM (due to extra $1/f$ suppression in the coupling⁷). The angle α is

$$\tan 2\alpha = \tan 2\beta \frac{m_A^2 + m_Z^2}{m_A^2 - m_Z^2}, \quad -\pi/2 \leq \alpha \leq 0 \quad (41)$$

If the lightest neutralino is light enough, $m_{\chi_1^0} < m_h$, then h^0, H^0 can decay into it and a goldstino which has a mass of order $f/M_{Planck} \sim 10^{-3}$ eV; if this is not the case, the decay of neutralino into h^0 and goldstino takes place, examined in [29]. In the former case, the partial decay rate is

$$\Gamma_{h^0 \rightarrow \chi_1^0 \psi_X} = \frac{m_h}{16\pi f^2} \left| \sum_{k=1}^4 \delta'_k \mathcal{Z}_{1k} \right|^2 \left(1 - \frac{m_{\chi_1^0}^2}{m_{h^0}^2} \right)^2 \quad (42)$$

The partial decay rate has corrections coming from both higgsino ($\mathcal{Z}_{13}, \mathcal{Z}_{14}$) and gaugino fields ($\mathcal{Z}_{11}, \mathcal{Z}_{12}$), since they both acquire a goldstino component, see eqs. (38). The gaugino correction arises after gaugino-goldstino mixing, SUSY and EW symmetry breaking, (as shown by m_{λ_i}, m_Z dependence in δ'_k) and was not included in previous similar studies [25, 28, 29].

The partial decay rate is presented in Figure 2 for various values of μ, m_A and $m_{\lambda_{1,2}}$ which are parameters of the model. A larger decay rate requires a light $\mu \sim \mathcal{O}(100)$ GeV, when the neutralino χ_1^0 has a larger higgsino component. At the same time an increase of m_h above the LEP bound requires a larger value for μ , close to $\mu \approx 700$ GeV if $\sqrt{f} \approx 1.5$ TeV, and $\mu \approx 850$ GeV if $\sqrt{f} \approx 2$ TeV, see Figure 1 (c). The results in Figure 2 show that the partial decay rate can be significant ($\sim 3 \times 10^{-6}$ GeV), if we recall that the total SM Higgs decay rate (for $m_h \approx 114$ GeV) is about 3×10^{-3} GeV, with a branching ratio of $h^0 \rightarrow \gamma\gamma$ of 2×10^{-3} , (Figure 2 in [26]). Thus the branching ratio of the process can be close to that of SM $h^0 \rightarrow \gamma\gamma$. The decay is not very sensitive to $\tan \beta$ (Figure 2 (b)), due to the extra contribution (beyond MSSM) from the quartic Higgs coupling. It would be interesting to analyze the above decay rate at the one-loop level, for a more careful comparison to SM Higgs decays rates.

An interesting coupling that is also present in the $1/f$ order is that of goldstino to Z_μ boson and to a neutralino. Depending on the relative mass relations, it can bring about a decay of Z_μ (χ_j^0) into χ_j^0 (Z_μ) and a goldstino, respectively. The relevant terms are

$$\begin{aligned} \mathcal{L}^{new} + \mathcal{L}_0^{onshell} \supset & -\frac{1}{4} \bar{\psi}_{h_1^0} \bar{\sigma}^\mu \psi_{h_1^0} (g_2 V_2^3 - g_1 V_1)_\mu + \frac{1}{4} \bar{\psi}_{h_2^0} \bar{\sigma}^\mu \psi_{h_2^0} (g_2 V_2^3 - g_1 V_1)_\mu \Big\} \\ & - \sum_{i=1}^2 \frac{m_{\lambda_i}}{\sqrt{2} f} \psi_X \sigma^{\mu\nu} \lambda_i^a F_{\mu\nu, i}^a + h.c. \end{aligned} \quad (43)$$

⁷The relation is $h_1^{0'} = H^0 \cos \alpha - h^0 \sin \alpha$, and $h_2^{0'} = H^0 \sin \alpha + h^0 \cos \alpha$.

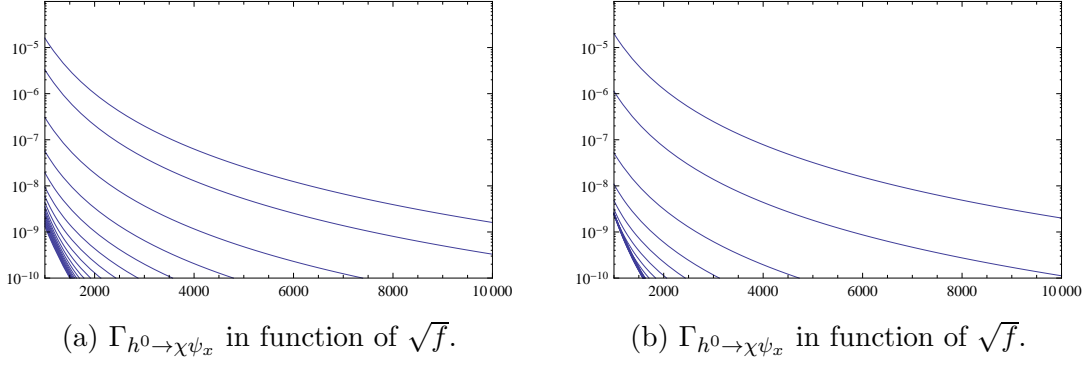


Figure 2: The partial decay rate of $h^0 \rightarrow \psi_X \chi_1^0$ for (a): $\tan \beta = 50$, $m_{\lambda_1} = 70$ GeV, $m_{\lambda_2} = 150$ GeV, μ increases from 50 GeV (top curve) by a step 50 GeV, $m_A = 150$ GeV. Compare against Figure 1 (c) corresponding to a similar range for the parameters. At larger μ , m_h increases, but the partial decay rate decreases. Similar picture is obtained at low $\tan \beta \sim 5$. (b): As for (a) but with $\tan \beta = 5$. Compare against Figure 1 (d). Note that the total SM decay rate, for $m_h \sim 114$ GeV, is of order 10^{-3} , thus the branching ratio in the above cases becomes comparable to that of SM Higgs going into $\gamma\gamma$ (see Figure 2 in [26]).

where the last term was generated in (25) (i labels the gauge group). Since the higgsinos acquired a goldstino component ($\propto \psi_X/f$) via mass mixing, the first line above induces additional $\mathcal{O}(1/f)$ couplings of the higgsino to goldstino and to $Z_\mu = (1/g)(g_2 V_2^3 - g_1 V_1)_\mu$ with $g^2 = g_1^2 + g_2^2$. After some calculations one finds the coupling $Z_\mu \chi_j^0 \psi_X$:

$$\mathcal{L}^{new} + \mathcal{L}_0^{onshell} = \frac{1}{f\sqrt{2}} \sum_{j=1}^4 \left[\bar{\psi}_X \bar{\sigma}^\mu \chi_j^0 Z_\mu (\mu m_Z w_j - m_Z^2 v_j) - \bar{\psi}_X (\bar{\sigma}^\mu \partial^\nu - \bar{\sigma}^\nu \partial^\mu) \chi_j^0 Z_{\mu\nu} v_j \right] + hc \quad (44)$$

where

$$w_j = \cos \beta \mathcal{Z}_{j4}^* - \sin \beta \mathcal{Z}_{j3}^*, \quad v_j = -\sin \theta_w \mathcal{Z}_{j1}^* + \cos \theta_w \mathcal{Z}_{j2}^*, \quad Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu \quad (45)$$

If $m_{\chi_1^0}$ is lighter than Z_μ then a decay of the latter into $\chi_1^0 + \psi_X$ is possible. The decay rate of this process is (with $j = 1$):

$$\Gamma_{Z \rightarrow \psi_X \chi_j^0} = \frac{m_Z^5}{32\pi f^2} \left[\zeta_1 |w_j|^2 + \zeta_2 |v_j|^2 + \zeta_3 (w_j v_j^* + w_j^* v_j) \right] \left(1 - \frac{m_{\chi_j}^2}{m_Z^2} \right)^2 \quad (46)$$

with $\zeta_1 = 2(2+r^2)\mu^2/m_Z^2$, $\zeta_2 = 2(8+r^2)(1+2r^2)$, $\zeta_3 = -2(4+5r^2)\mu/m_Z$ where $r = m_{\chi_j}/m_Z$ (in (44) and subsequent one can actually replace μ by m_{χ_j} and $w_j \rightarrow w_j^*$, with $\mathcal{Z}_{j4} \leftrightarrow \mathcal{Z}_{j3}$).

The decay rate should be within the LEP error for Γ_Z , which is 2.3 MeV [30] (ignoring theoretical uncertainties which are small). From this, one finds a lower bound for \sqrt{f} , which

can be as high as $\sqrt{f} \approx 700$ GeV for the parameter space considered previously in Figure 1, while generic values are $\sqrt{f} \sim \mathcal{O}(400)$ GeV. Therefore the results for the increase of m_h , that needed a value for \sqrt{f} in the TeV region, escape this constraint. This constraint does not apply if the lightest neutralino has a mass larger than m_Z , when the opposite decay ($\chi_j \rightarrow Z \psi_X$) takes place (this can be arranged for example by a larger m_{λ_1}).

There also exists the interesting possibility of an invisible decay of Z_μ gauge boson into a pair of goldstino fields, that we review here [6, 13, 15]. This is induced by the following terms in the Lagrangian, after the Higgs field acquires a VEV:

$$\begin{aligned} \mathcal{L}^{new} + \mathcal{L}_0^{onshell} \supset & \left\{ \frac{1}{4f^2} \bar{\psi}_X \bar{\sigma}^\mu \psi_X (g_2 V_2^3 - g_1 V_1)_\mu (m_1^2 v_1^2/2 - m_2^2 v_2^2/2) \right. \\ & \left. - \frac{1}{4} \bar{\psi}_{h_1^0} \bar{\sigma}^\mu \psi_{h_1^0} (g_2 V_2^3 - g_1 V_1)_\mu + \frac{1}{4} \bar{\psi}_{h_2^0} \bar{\sigma}^\mu \psi_{h_2^0} (g_2 V_2^3 - g_1 V_1)_\mu \right\} + h.c. \end{aligned} \quad (47)$$

With (38) and (47) one finds the coupling of Z boson to a pair of goldstinos:

$$\mathcal{L}^{new} + \mathcal{L}_0^{onshell} \supset \frac{m_Z^2}{4f^2} \bar{\psi}_X \bar{\sigma}^\mu \psi_X Z_\mu \langle D_Z \rangle + h.c. \quad (48)$$

where $\langle D_Z \rangle \equiv \cos \theta_W \langle D_2^3 \rangle - \sin \theta_W \langle D_1 \rangle = -(m_Z^2/g) \cos 2\beta + \mathcal{O}(1/f)$. The decay rate is then

$$\Gamma_{Z \rightarrow \psi_X \psi_X} = \frac{m_Z}{24\pi g^2} \left[\frac{m_Z^4}{2f^2} \right]^2 \cos^2 2\beta \quad (49)$$

in agreement with previous results obtained for $B = 0$ [6, 13, 15]. The decay rate is independent of m_A and should be within the LEP error for Γ_Z (2.3 MeV [30]). One can then easily see that the increase of the Higgs mass above the LEP bound (114.4 GeV) seen earlier in Figure 1 is consistent with the current bounds for this decay rate, which thus places only mild constraints on f , below the TeV scale (≈ 200 GeV) [6, 15].

Similarly, \mathcal{L}^{new} can also induce Higgs decays into goldstino pairs. The terms in \mathcal{L}^{new} that contribute to Higgs decays are $\mathcal{L}_{F(2)}^{aux}$, \mathcal{L}_D^{aux} , \mathcal{L}_m^{extra} together with the MSSM higgsino-Higgs-gaugino coupling (last term in (37)). After using (38), expanding the Higgs fields about their VEV, one finds:

$$\mathcal{L}^{new} + \mathcal{L}_0^{onshell} \supset \frac{\mu v}{4f^2} m_A^2 \cos 2\beta \bar{\psi}_X \bar{\psi}_X \left[h_1^{0'} \sin \beta - h_2^{0'} \cos \beta \right] + h.c. + \mathcal{O}(1/f^3) \quad (50)$$

which, similarly to Z couplings, is independent of gaugino masses. Here $v = 246$ GeV and $h_i^0 = 1/\sqrt{2} (v_i + h_i^{0'} + i\sigma_i)$, $\langle h_i^{0'} \rangle = 0$, $\langle \sigma_i \rangle = 0$. In the mass eigenstates basis one simply replaces the square bracket in (50) by $[H^0 \sin(\beta - \alpha) - h^0 \cos(\beta - \alpha)]$. One can also replace

m_A by $m_A^2 = m_h^2 + m_H^2 - m_Z^2 + \mathcal{O}(1/f^2)$, where the Higgs masses can be taken to be the MSSM values (up to higher order corrections in $1/f$). The decay rate of h^0 into a pair of goldstinos is then

$$\Gamma_{h^0 \rightarrow \psi_X \psi_X} = \frac{m_h}{8\pi f^4} g_{h^0 \psi_X \psi_X}^2 \quad (51)$$

where $g_{h^0 \psi_X \psi_X}$ is the coupling of $h^0 \psi_X \psi_X$ of the above Lagrangian. For relevant values of f above ~ 1 TeV it turns out that this decay rate is very small relative to other partial decay rates of the Higgs in the MSSM/SM. For example, for a total decay rate near 10^{-3} GeV (valid near a Higgs mass of order $\mathcal{O}(100)$ GeV), the branching ratio of this decay mode is well below the usual ones and below that of SM Higgs going into $\gamma\gamma$, by a factor $\approx 10^{-3} - 10^{-2}$.

7 Conclusions

In this work we performed a model independent analysis of the consequences of a light goldstino (of mass $\sim f/M_{Planck}$) and investigated its couplings to the MSSM superfields. This was done by treating \sqrt{f} as a free parameter that can be as low as few times the soft SUSY breaking scale $m_{soft} \sim \text{TeV}$. The formalism parametrized but did not predict the soft masses, assumed to be fixed (near the TeV scale) by an otherwise arbitrary SUSY breaking sector.

The goldstino couplings can be determined by non-linear supersymmetry. Above the m_{soft} scale, one has the usual MSSM superfields and the goldstino couples to them, while below this scale the SM superpartners are integrated out and one is left with the goldstino coupled to SM fields. Both these cases can actually be treated using constrained superfields, where the constraints effectively integrate out the corresponding superpartners in terms of light degrees of freedom. For energy regimes $E \sim m_{soft} \leq \sqrt{f}$ the only constrained superfield is that of goldstino, which couples to the MSSM superfields via the soft terms. Below this energy regime additional constraints should be imposed on the MSSM superfields themselves. If supersymmetry breaking scale is low $\sqrt{f} \sim \text{few TeV}$, the goldstino couplings to the MSSM become important. In this paper the leading couplings of all MSSM fields to the goldstino were computed to $1/f^2$ order, and these can be used for phenomenological studies. In the limit the hidden sector SUSY scale is large with fixed soft masses, the MSSM with explicit soft breaking terms is recovered.

A significant impact of the aforementioned couplings turned out to be in the Higgs sector of the MSSM. It was noticed that the usual MSSM scalar potential acquires additional terms involving Higgs quartic couplings, with coefficients depending on the ratio of the soft masses to the ‘hidden’ sector SUSY breaking scale (\sqrt{f}). The presence of these couplings, effectively generated by integrating out the sgoldstino via its superfield constraint, can have a significant

impact for the Higgs mass and electroweak scale fine-tuning of the MSSM and these were investigated in detail.

The masses of the CP even and CP odd higgses were computed to order $1/f^2$. It was shown that for a low scale of SUSY breaking, the SM-like Higgs mass m_h is increased, to reach and cross the LEP bound, already at the tree level. For values of \sqrt{f} between 1.5 TeV to 7 TeV one obtains a value of m_h above the LEP bound. The correction increases with μ and can remain significant even above this energy range. As in the MSSM, quantum corrections increase m_h further. The quartic Higgs coupling was also increased by an additional (effective) contribution related to the Higgs soft terms. The benefit of this is that the amount of fine tuning of the electroweak scale is then reduced relative to that in the MSSM alone, by a factor comparable to (or even larger) than that due to the MSSM quantum corrections to the quartic Higgs coupling. This can be easily understood if we recall that the main source of fine tuning in the MSSM is related to the smallness of the MSSM Higgs quartic coupling.

The mechanism by which the mass of the SM-like Higgs is increased and the fine-tuning reduced has similarities with the method of additional effective operators usually considered in the MSSM Higgs sector, to solve these problems. Indeed, using effective operators of dimension $d = 5$ suppressed by “new physics” at a scale M , one can increase the Higgs mass above the LEP bound. The advantage in our case is that no new scale is introduced in the visible sector. In both cases the new scales introduced (M or \sqrt{f}) have comparable values, because in both cases the required increase for m_h to be above the LEP bound is done via couplings that depend on the ratio μ/M and μ/\sqrt{f} , respectively.

The possibility of an invisible decay of the MSSM lightest Higgs or of Z boson into a goldstino and the (lightest) neutralino was investigated. The decay rate of the Higgs can become comparable to the $h^0 \rightarrow \gamma\gamma$ partial decay mode while that of the Z boson was shown to bring a lower bound on $\sqrt{f} \sim 700$ GeV, which is stronger than previous similar bounds on \sqrt{f} . This bound is consistent with that required for a classical increase of m_h above the LEP bound, and does not apply in the case $m_{\chi_1^0} > m_Z$ (if for example m_{λ_1} is large enough). Higher order (in $1/f$) processes such as Z or Higgs decay into pairs of goldstinos were also analyzed; these were found to be too small to bring constraints on \sqrt{f} (for Z case) or sub-leading to the decay into goldstino-neutralino (for the Higgs case), and in agreement with previous calculations.

Let us mention that although we treated all MSSM superfields in the linear SUSY realization (*i.e.* squarks and sleptons lighter than \sqrt{f}), our results on Higgs mass and invisible decays of the Higgs and Z bosons are largely independent of this assumption. Even if the quarks and leptons superfields are treated in the nonlinear SUSY realization (*i.e.* squarks and slepton masses are large enough to be integrated out), these results are not changed.

Acknowledgments

This work was supported in part by the European Commission under contracts PITN-GA-2009-237920, MRTN-CT-2006-035863 and ERC Advanced Grant 226371 (“MassTeV”), by INTAS grant 03-51-6346, by ANR (CNRS-USAR) contract 05-BLAN-007901, by CNRS PICS no. 3747 and 4172. During the last stages of this work, it was also partially supported by the U.S. National Science Foundation under Grant No. PHY05-51164. I.A. and E.D. would like to thank the KITP of UC Santa Barbara for hospitality during the last part of this work. P.T. would like to thank the ‘Propondis’ Foundation for its support. D.G. thanks the CERN Theory Division for the financial support. D.G. also thanks S. P. Martin, Z. Komargodski and N. Seiberg for clarifications on goldstino couplings in the constrained superfield formalism and their implications for phenomenology.

Appendix

From the Higgs part of the scalar potential, eq. (17), one finds the exact (in $1/f$) form of the masses of the CP even Higgs fields:

$$\begin{aligned}
m_{h,H}^2 &= -2\mu^2 \mp \frac{1}{4}\sqrt{\sigma} + \frac{-2B}{8\sin 2\beta} \left[3 + \sqrt{w_0} \right] + \frac{4\mu^4 - B^2 + 2\mu^2 m_Z^2 + B m_Z^2 \sin 2\beta}{4\mu^2 + 2m_Z^2 \cos^2 2\beta + 2B \sin 2\beta} \\
&+ (1 - \sqrt{w_0}) \frac{2f^2}{v^2} \frac{B^2 + 4\mu^4 + m_Z^2(4\mu^2 + m_Z^2) \cos^2 2\beta + 4B\mu^2 \sin 2\beta}{(4\mu^2 + 2m_Z^2 \cos^2 2\beta + 2B \sin 2\beta)^2}
\end{aligned} \tag{52}$$

with the upper sign for the lightest m_h . The following notation was used

$$w_0 \equiv 1 - \frac{v^2}{f^2} (4\mu^2 + 2m_Z^2 \cos^2 2\beta + 2B \sin 2\beta) \tag{53}$$

and

$$\begin{aligned}
\sigma &= 2 \left[2x_1^2 + 8B^2 + 5m_Z^4 + m_Z^2 (8x_1 \cos 2\beta + 3m_Z^2 \cos 4\beta - 8B \sin 2\beta) \right] + \frac{v^4}{2f^4} \left[2x_2^4 + 5B^4 \right. \\
&+ 20x_2^2 x_1^2 + 5x_1^4 + 12x_1 x_2 (2x_1^2 + 2B^2 + x_2^2) \cos 2\beta + 3(x_1^2 - B^2)(x_1^2 + B^2 + 2x_2^2) \cos 4\beta \\
&+ 10B^2 (x_1^2 + 2x_2^2) + 12B(2(x_1^2 + B^2)x_2 + x_2^3 + x_1(x_1^2 + B^2 + 2x_2^2) \cos 2\beta) \sin 2\beta \Big] \\
&+ \frac{2v^2}{f^2} \left[-4B^2(m_Z^2 - 3x_2) + m_Z^2 x_2^2 + x_1^2(5m_Z^2 + 6x_2) + 3m_Z^2(x_1^2 + x_2^2) \cos 4\beta \right. \\
&+ 2x_1(2x_1^2 + 5B^2 + x_2(6m_Z^2 + x_2)) \cos 2\beta + 2B(x_1^2 + 4B^2 - 3m_Z^2 x_2 + 2x_2^2 \\
&+ 3x_1 m_Z^2 \cos 2\beta) \sin 2\beta \Big], \quad \text{with} \quad x_1 \equiv m_1^2 - m_2^2, \quad x_2 \equiv m_1^2 + m_2^2.
\end{aligned} \tag{54}$$

At the minimum of the scalar potential $x_{1,2}$ take the values given in the text, Section 5. A series expansion (in $1/f$) of $m_{h,H}^2$ was presented in the text.

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